LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - MATHEMATICS

FOURTH SEMESTER – APRIL 2013

MT 4960 - FLUID DYNAMICS

Date : 06/05/2013   Dept. No.   Max. : 100 Marks
Time : 1:00 - 4:00

Answer **ALL** Questions. All question carry equal marks.

1. (a) (i) Define density, pressure, compressibility and viscosity of fluid flow.   (5)

   OR

   (ii) Derive the equation of stream lines.   .   (5)

   (b) (i) If the velocity of an incompressible fluid at the point (x, y, z) is given by $\left(\dfrac{3xz}{r^5}, \dfrac{3yz}{r^5}, \dfrac{3z^2 - r^2}{r^5} \right)$

   where $r^2 = x^2 + y^2 + z^2$, show that the fluid motion is possible and velocity potential is $\dfrac{cos\theta}{r^2}$.
   Find the equation of streamlines. (15)

   OR

   (ii) Test whether the motions specified by $\vec{q} = k^2 \dfrac{\left(x \vec{j} - y \vec{i} \right)}{x^2 + y^2}$, where k is a constant is a

   possible fluid motion for a incompressible fluid. If so find the equation of stream lines. Also test whether the motion is of potential kind and if so determine the velocity potential.

   (iii) Derive Material, Local and Connective derivatives.   (8+7)

2. (a) (i) For a steady fluid motion the velocity is given by $u = \dfrac{ax - by}{x^2 + y^2}, v = \dfrac{ay + bx}{x^2 + y^2}, w = 0$. Find the pressure

   at any point.   (5)

   OR

   (ii) Derive Euler's equation of motion.   (5)

   (P.T.O)

   (b) (i) The particle velocity for a liquid motion refer to a rectangular axes is given by the components

   $u = A \cos \dfrac{\pi x}{2a} \cos \dfrac{\pi z}{2a}$, $v = 0$, $w = A \sin \dfrac{\pi x}{2a} \sin \dfrac{\pi z}{2a}$, where A is a constant. Show that this is a

   possible motion of incompressible fluid under no body force in an infinite fixed rigid tube. Also find the pressure associated with this velocity.

   (ii) A stream is rushing from a boiler through a conical pipe, the diameter of   ends of which is D and d respectively. If V and v are the velocities of the stream and if the motion be supposed to be that of divergence from the vertex of the cone.   Prove that

   $\dfrac{v}{V} = \dfrac{D^2}{d^2} \exp \left(\dfrac{v^2 - V^2}{2k} \right)$, where k is the pressure divided by density.

   (8+7)

   OR

   (iii) Derive Kelvin's circulation theorem.

(iv) A steady fluid motion is given by the velocity vector $\vec{q} = x\,\vec{i} + y\,\vec{j}$ and the external force is gravity. Find the pressure at any point and the surface of equal pressure. (9+6)

3.(a) (i) Briefly explain doublet of a stream.

OR

(ii) Briefly explain source and sink with an example.   (5)

(b) (i) If there is a source at $(a,0)$ and $(-a,0)$, sink at $(0,a)$ and $(0,-a)$ and are of equal strength. Show that the circle through which the four points passes is a stream line.

(15)

OR

(ii) Explain the construction and working of a Pitot tube and Venturi tube. (15)

4. (a) (i) State and prove Butler sphere theorem.   (5)

OR

(ii) Write notes on Aerofoil.   (5)

(P.T.O)

(b) (i) State and prove the theorem of Kutta-Joukowski.   (15)

OR

(ii) Derive Stroke's stream function.   (15)

5. (a) (i) Find the components of force for an infinite circular cylinder in a uniform stream with circulation.   (5)

OR

(ii) Discuss the flow through a tube having uniform elliptic cross-section.   (5)

(b) (i) Discuss the steady motion tube having equilateral triangular cross section. (15)

OR

(ii) Derive the Navier-Stokes equation of motion for viscous fluid.   (15)

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